SUMMARY OF THE SYLLABUS FOR THE RE-EXAM IN MATH 001

1. Natural numbers

The numbers 1, 2, 3, ….. are called the natural numbers. The sum and product of two natural numbers are again natural where as the difference and quotient of two natural numbers need not be natural.

2. Fractions & Operations with fractions

The numbers of the form $\frac{a}{b}$ where a and b are natural numbers are called the **fractions**. Here, a is called the numerator and b is called the denominator. A fraction whose numerator is smaller than the denominator is called a **proper** fraction while a fraction whose numerator is bigger than the denominator is called an **improper** fraction. When we interchange the numerator and the denominator of a fraction, we get its **reciprocal.** Two or more fractions with same denominators are called **like** fractions. If we divide or multiply the numerator and the denominator of a fraction by a natural number, we get an **equivalent** fraction to the given fraction. A fraction that cannot be further simplified (i.e, if there is no common factor for the numerator and the denominator) is said to be in its **simplest form** or **lowest form.**

To arrange the fractions in the **ascending** order means to arrange them from smaller to bigger. The fractions that are arranged from bigger to smaller are said to be in the **descending order.**

a) *Addition of fractions*

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a}{b} + \frac{c}{d} + \frac{e}{f} = \frac{adf+cbf+ebd}{bdf}$$

b) *Subtraction of fractions*

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

c) *Multiplication of fractions*

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

d) *Division of fractions*

In order to divide a fraction by another fraction, multiply the first fraction by the reciprocal of the second.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

3. The integers

The natural numbers, the negatives of natural numbers and zero are called the **integers.**
Thus the integers are …., - 3, - 2, - 1, 0 , 1, 2, 3, …….
The sum, difference and the product of two integers are again integers; but the quotient of two integers need not be an integer.

We use the following rules while dealing with the operations on integers.

a) Addition:

(+) + (+) = (+)
(-) + (-) = (-)
(+) + (-) = *find the difference of the numbers and put the sign of the bigger one*
(-) + (+) = *find the difference of the numbers and put the sign of the bigger one*

b) Subtraction:

To subtract a positive number, we add the corresponding negative number and to subtract a negative number, we add the corresponding positive integer.

For eg:	2 – 5	= 2 + (- 5)	= ***- 3***
	-3 – 6	= -3 + (- 6)	= ***- 9***
	5 – (- 3)	= 5 + 3	= ***8***
	- 7 – (- 4)	= - 7 + 4	= ***- 3***

c) Multiplication:

$(+) \times (+) = (+)$
$(-) \times (-) = (+)$
$(+) \times (-) = (-)$
$(-) \times (+) = (-)$

d) Division:

$(+) \div (+) = (+)$
$(-) \div (-) = (+)$
$(+) \div (-) = (-)$
$(-) \div (+) = (-)$

Absolute value (Modulus) of a number:

The absolute value or modulus of a number is the distance of that number from zero.
Modulus of 5 is written by | 5 | and it is equal to 5.
Also, | - 4 | = 4.

4. Rational Numbers

The numbers of the form $\frac{p}{q}$ where p and q are integers with $q \neq 0$ are called **rational numbers.** Eg: $\frac{1}{2}, \frac{-2}{3}, \frac{-3}{-5}, \frac{3}{-4}$, - 6, 0 , 2, 3.5

Note: For addition, subtraction, multiplication and division, we use rules in fractions and integers.

5. Algebraic expressions

A combination of constants and variables connected by some or all of the four fundamental operations +, – , × and ÷ is called an **algebraic expression**.
The different parts of the algebraic expression separated by the sign + or – are called the ***terms*** of the expression.

Eg: (i) $5 - 3x + 4x^2y$ is an algebraic expression consisting of three terms, namely 5, $-3x$ and $4x^2y$

(ii) $7x^2 - 5xy + y^2z - 8$ is an algebraic expression consists of four terms, namely $7x^2$, $-5xy$, y^2z and -8.

Each term of an algebraic expression consists of a product of constants and variables. The number present in each term is called the ***numerical coefficient*** (or ***coefficient*** in short). In the term $5x^2y$, the coefficient is 5 and in the term x^3y, the coefficient is 1.

A term without any variables is called the *constant term*. In the expression, $3a + 2b - 5$, the constant term is -5.

Two or more terms of an algebraic expression are said to be **like terms** if

(i) they have the same variables and
(ii) the exponents (powers) of each variable are the same.

Otherwise, we call them as *unlike terms.*

The sum of several like terms is another like term whose coefficient is the sum of the coefficients of those like terms.

Eg: (i) $-a$, $7a$ and $5a$ are like terms. Moreover,
$(-a) + (7a) + (5a) = (-1 + 7 + 5)a = 11a$

(ii) The terms $4a^2b^3$, $7b^3a^2$ and $5a^2b^3$ are all like terms.
$4a^2b^3 + 7b^3a^2 + 5a^2b^3 = (4 + 7 + 5)\, a^2b^3 = 16a^2b^3$

(iii) The terms x^3y^2 and x^2y^3 are unlike as the powers of the variables are different.

Worked Examples:

1. Add: $5x^2 - 7x + 3$, $-8x^2 + 2x - 5$ and $7x^2 - x - 2$

Solution: Required sum

$$= (5x^2 - 7x + 3) + (-8x^2 + 2x - 5) + (7x^2 - x - 2)$$
$$= \underbrace{\left(5x^2 - 8x^2 + 7x^2\right) + \left(-7x + 2x - x\right) + \left(3 - 5 - 2\right)}_{\textit{collecting like terms}}$$
$$= (5 - 8 + 7)x^2 + (-7 + 2 - 1)x + (3 - 5 - 2)$$
$$= \mathbf{4x^2 - 6x - 4}$$

2. Add: $x^2 + xy + y^2 + xy^2$ and $-x^2 + 2yx - 3y^2 - x^2y$

Solution: Required sum

$$= (x^2 + xy + y^2 + xy^2) + (-x^2 + 2yx - 3y^2 - x^2y)$$

$= (x^2 - x^2) + (xy + 2yx) + (y^2 - 3y^2) + xy^2 - x^2y$
$= \mathbf{3xy - 2y^2 + xy^2 - x^2y}$

3. Simplify: $(2x^3 - 2x^2 - 2) - (2x^3 - 2x - 1) - (2x^3 - x^2 - x + 1)$

Solution: $(2x^3 - 2x^2 - 2) - (2x^3 - 2x - 1) - (2x^3 - x^2 - x + 1)$
$= 2x^3 - 2x^2 - 2 - 2x^3 + 2x + 1 - 2x^3 + x^2 + x - 1$
$= (2x^3 - 2x^3 - 2x^3) + (-2x^2 + x^2) + (2x + x) + (-2 + 1 - 1)$
$= \mathbf{- 2x^3 - x^2 + 3x - 2}$

6. Exponents

The number $\underbrace{a \times a \times a \times \ldots \times a}_{b \ \ times}$ can be written as a^b and can be read as '*a raised to b'.* Here we call *a* as the **base** and *b* as the **exponent** (or **power**). This notation is called **exponential form** or **power notation.**

Similarly, a rational number multiplied several times can be expressed in the same notation.
For eg. $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$.

The reciprocal of a rational number $\frac{a}{b}$ can be expressed as $\left(\frac{a}{b}\right)^{-1}$ and we know that

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

<u>Laws of Exponents</u>

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$ or $\frac{a^m}{a^n} = a^{m-n}$
3. $\left(a^m\right)^n = a^{mn}$
4. $(ab)^m = a^m b^m$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
6. $a^{-n} = \frac{1}{a^n}$ In particular, $a^{-1} = \frac{1}{a}$
7. $\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$
8. For any non-zero real number *a,* $a^0 = 1$

Worked Examples:

1. Divide: $\dfrac{24xyz^3}{-3z^2}$

Solution: $\dfrac{24xyz^3}{-3z^2} = -8xyz$

2. Simplify: $\dfrac{20^6}{25^5}$

Solution: $\dfrac{20^6}{25^5} = \left(\dfrac{20}{25}\right)^5 \times 20 = \left(\dfrac{4}{5}\right)^5 \times 20$

$$= \frac{4^5 \times 20}{5^5} = \frac{4^5 \times 4}{5^4} = \frac{4^6}{5^4} = \frac{4096}{625}$$

3. Show that: $\left(a^x b^y\right)\left(\dfrac{b^{2x}}{a^{-y}}\right) = a^{x+y}b^{y+2x}$

Solution: $\left(a^x b^y\right)\left(\dfrac{b^{2x}}{a^{-y}}\right) = a^x b^y . a^y b^{2x}$

$$= a^x a^y b^y b^{2x}$$
$$= a^{x+y}b^{y+2x}$$

4. Simplify : $\left(4^{-1} + 8^{-1}\right) \div \left(\dfrac{2}{3}\right)^{-1}$

Solution: $\left(4^{-1} + 8^{-1}\right) \div \left(\dfrac{2}{3}\right)^{-1} = \left(\dfrac{1}{4} + \dfrac{1}{8}\right) \div \left(\dfrac{3}{2}\right)$

$$= \left(\frac{2}{8} + \frac{1}{8}\right) \div \left(\frac{3}{2}\right)$$
$$= \frac{3}{8} \div \frac{3}{2} = \frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$$

5. Evaluate $\left\{\left(\dfrac{1}{3}\right)^{-3} - \left(\dfrac{1}{2}\right)^{-3}\right\} \div \left(\dfrac{1}{4}\right)^{-3}$

Solution: $\left\{\left(\dfrac{1}{3}\right)^{-3} - \left(\dfrac{1}{2}\right)^{-3}\right\} \div \left(\dfrac{1}{4}\right)^{-3} = \left\{3^3 - 2^3\right\} \div 4^3$

$$= \left\{27 - 8\right\} \div 64$$
$$= 19 \div 64$$
$$= \frac{19}{64}$$

7. Algebraic identities

1. $a(b + c) = ab + ac$
2. $(a + b)(c + d) = ac + ad + bc + bd$
3. $(a + b)^2 = a^2 + 2ab + b^2$
4. $(a - b)^2 = a^2 - 2ab + b^2$
5. $(a + b)(a - b) = a^2 - b^2$

8. Term by term Division

To divide an expression $a + b$ by another expression c, we need to divide each term of $a + b$ by c.

$$\frac{a+b}{c} = \frac{a}{c}+\frac{b}{c}$$

Do not get confused the expression $\frac{a+b}{c}$ with one of the form $\frac{ab}{c}$. An expression of the form $\frac{ab}{c}$ can be evaluated in many ways. For eg, one may evaluate the product ab first and divide it by c or divide a by c and multiply it by b *etc*. Please see the following examples.

1. Divide: $\frac{3x^2 - 12xy}{-x} = \frac{3x^2}{-x} - \frac{12xy}{-x} = -3x + 12y$
2. Divide: $\frac{(3x^2)(-12xy)}{-x} = \frac{-36x^3y}{-x} = 36x^2y$

9. Factoring

To factorize an expression means to express it as a product. Here we consider the expressions of the form $ab+ac$ and $ac+ad+bc+bd$

To factorize $ab + ac$, we need to find the common factors of the terms. Thus,

$ab + ac = a(b + c)$

To factorize expressions of the fom $ac+ad+bc+bd$, we need to consider the terms as groups of $ac + ad$ and $bc + bd$ separately and proceed as above.

Thus, $ac+ad+bc+bd = (ac + ad) + (bc + bd)$ [*Consider as groups of two*]

$= a(c+ d) + b(c + d)$ [*Factorize each group*]

$=\mathbf{(c + d)(a + b)}$ [*Factor*(*c*+*d*) *from both*]

10. Factoring the expressions of the form $x^2 + px + q$

To factorize the expressions of the from $x^2 + px + q$, we need to find two numbers a and b such that $a + b = p$ and $ab = q$. Then $x^2 + px + q = (x + a)(x + b)$

Eg: $x^2 + 5x + 6 = (x + 2)(x + 3)$ [a+b = 5, ab = 6. ∴ a = 2, b = 3]

$x^2 - 5x + 6 = (x - 2)(x - 3)$ [a+b = -5, ab = 6. ∴ a = -2, b = -3]

$x^2 + 5x - 6 = (x + 6)(x - 1)$ [a+b = 5, ab = -6. ∴ a = 6, b = -1]

$x^2 - 5x - 6 = (x - 6)(x + 1)$ [a+b = -5, ab = -6. ∴ a = -6, b = 1]

11. Linear Equations in one variable

Linear equations in one variable are equations of the form $ax + b = 0$. To solve an equation means to find the value of the variable that satisfies the equation and that value is called *solution* or *root* of the equation.

Worked Examples:

1. Solve: $3x - 4 = 5$

Solution: $3x - 4 = 5 \Rightarrow 3x = 5+4$

$\Rightarrow 3x = 9$

$\Rightarrow x = \frac{9}{3}$

$\Rightarrow \mathbf{x = 3}$

2. Solve: $\frac{2z-1}{3} = \frac{z+1}{4}$

Solution: $\frac{2z-1}{3} = \frac{z+1}{4} \Rightarrow 4(2z - 1) = 3(z + 1)$

$\Rightarrow 8z - 4 = 3z + 3$

$\Rightarrow 8z - 3z = 3+4$

$\Rightarrow 5z = 7$

$\Rightarrow z = \frac{7}{5}$

SAMPLE QUESTION PAPER

(Note that the following is a specimen question paper for the exam. Answers and the hints for all the questions are given at the end. You may use this to have more focus on the important areas of the syllabus. However, the final exam will be different from this.)

1. Evaluate: $3^2 + (5^2 - 4^2)$

(A) -18 (B) 1 (C) 18 (D) 0

2. The smallest natural number is

(A) 0 (B) 1 (C) 2 (D) 3

3. Convert $\frac{12}{36}$ into a fraction with denominator 6

(A) $\frac{2}{6}$ (B) $\frac{3}{6}$ (C) $\frac{1}{6}$ (D) $\frac{5}{6}$

4. Evaluate: $\frac{20}{9} \div \frac{1}{18}$

(A) 10 (B) 40 (C) $\frac{20}{81}$ (D) $\frac{81}{20}$

5. Find: $(-100) - (-100) + |-100|$

(A) 300 (B) -100 (C) -300 (D)100

6. Which of the following is not a fraction?

(A) 3 (B) -3 (C) 3.5 (D) $\frac{1}{6}$

7. The proper fraction of the following is

(A) $\frac{7}{6}$ (B) $\frac{11}{6}$ (C) $\frac{1}{6}$ (D) $\frac{10}{6}$

8. Evaluate $x^2 + y^2$ if $x = -\frac{2}{3}$ and $y = \frac{3}{5}$

(A) $\frac{-181}{225}$ (B) $\frac{19}{225}$ (C) $\frac{-19}{225}$ (D) $\frac{181}{225}$

9. In the algebraic expression $3x^3 - 2x^2 + 4x - 7$, the coefficient of x^2 is

(A) 2 (B) -2 (C) 3 (D) 4

10. Simplify: $a(b - c) + b(c - a) + c(a - b)$

(A) a-c (B) 0 (C) a-b (D) 1

11. Simplify: $\frac{24ab^3}{6ab}$

(A) $4ab$ (B) $4ab^2$ (C) $4b^2$ (D) $4a^2b^4$

12. Simplify: $\frac{x^9 y^{-12}}{x^{-6} y^8}$

(A) $x^{15}\ y^{-20}$ (B) $x^{-15}\ y^{-20}$ (C) $x^{-15}\ y^{20}$ (D) $x^{15}\ y^{20}$

13. Simplify $\left(\frac{1}{2}\right)^{-3}$

(A) 8 (B) -8 (C) 2^{-3} (D) $\left(\frac{1}{2}\right)^3$

14. Simplify $(a + 2)^2 - (a - 2)^2$

(A) 8a (B) -8a (C) $2a^2 + 8a$ (D) $2a^2 + 8$

15. Divide : $\frac{-2m^2 - 3mn}{-m}$

(A) 2m + 3n (B) 2m– 3n (C) -2m + 3n (D) -2m -3n

16. Simplify: $\frac{(-4x^6)(-2x^4)}{-8x^{10}}$

(A) 1 (B) $\frac{1}{2x^4} + \frac{1}{4x^6}$ (C) - 1 (D) $\frac{-2}{x^3}$

17. Evaluate $\frac{(x^2)(-xy)(-xz)}{-x}$

(A) $-x^3yz$ (B) x^3yz (C) x^2yz (D) $-x^2yz$

18. Factorize : $15x^2 y^2 - 9xy^3$

(A) $3xy(5x - 3y)$ (B) $3x^2y^2(5x - 3y)$ (C) $3xy(5x^2 - 3y)$ (D) $3xy^2(5x - 3y)$

19. Factorize: $ab + a^2b + ab^2$

(A) $a(b + ab + b^2)$ (B) $b(a + a^2 + ab)$ (C) $ab(1 + a + b)$ (D) $ab(a + b)$

20. Factorize: $2a - 2b - ax + bx$

(A) $(a-b)\ (2+x)$ (B) $(a-b)\ (-2-x)$

(C) $(a-b)\ (-2+x)$ (D) $(a-b)\ (2-x)$

21. Factorize: $x^2 + x - 12$

(A) $(x+3)(x-4)$ (B) $(x-3)(x-4)$

(C) $(x+3)(x+4)$ (D) $(x-3)(x+4)$

22. Factorize: $x^2 - 3x - 10$

(A) $(x+5)(x+2)$ (B) $(x-5)(x+2)$

(C) $(x-5)(x-2)$ (D) $(x+5)(x-2)$

23. Find x: $2x-3=5$

(A) -4 (B) 4 (C) 1 (D) -1

24. Find x: $\frac{x}{2} = -22$

(A) 44 (B)11 (C) – 44 (D) – 11

25. Find the value of z: $3z - 5 = 28$

(A) 10 (B) 11 (C) $\frac{23}{3}$ (D) $-\frac{23}{3}$

26. Solve for x: $\frac{3}{x-2} = \frac{2}{x+3}$

(A) 13 (B) 5 (C) -13 (D) -5

27. Arrange the following fractions in the ascending order:

$\frac{3}{7}, \quad \frac{2}{7}, \quad \frac{5}{7}$

(A) $\frac{3}{7}, \frac{5}{7}, \frac{2}{7}$ (B) $\frac{5}{7}, \frac{3}{7}, \frac{2}{7}$

(C) $\frac{2}{7}, \frac{3}{7}, \frac{5}{7}$ (D) $\frac{5}{7}, \frac{2}{7}, \frac{3}{7}$

28. Factorize: $18a^3b^2 - 27a^2b^3$

(A) $9a^2b^3(2a - 3b))$ (B) $9a^2b^2(2a - 3b)$

(C) $9a^2b(2a - 3b)$ (D) $9a^3b^2(2a - 3b)$

29. How many terms are there in the algebraic expression $3x^2y$?

(A) 3 (B) 2 (C) 1 (D) 4

30. Simplify: $\dfrac{(12x^2y^3)(6x^4y^2)}{3x^3y^3}$

(A) $24x^3y^2$ (B) $\dfrac{4}{x}-\dfrac{2x}{y}$ (C) $\dfrac{4}{x}+\dfrac{2x}{y}$ (D) $-24x^3y^2$

ANSWERS AND HINTS

Qn No.	Correct Choice	Answers (Hints)
1	**C**	$3^2 + (5^2 - 4^2) = 9 + (25 - 16) = 9 + 9 =$ **18**
2	**B**	Natural numbers are 1, 2, 3, ……
3	**A**	$\dfrac{12}{36}=\dfrac{12\div 6}{36\div 6}=\dfrac{2}{6}$
4	**B**	$\dfrac{20}{9}\div\dfrac{1}{18}=\dfrac{20}{9}\times\dfrac{18}{1}=20\times 2=40$
5	**D**	(-100) - (- 100) + |-100| = - 100 + 100 + 100 = 100
6	**B**	A fraction cannot be negative.
7	**C**	A proper has its numerator smaller than denominator
8	**D**	$x^2 + y^2 = (-\dfrac{2}{3})^2 + (\dfrac{3}{5})^2 = \dfrac{4}{9}+\dfrac{9}{25} = \dfrac{4\times 25+9\times 9}{9\times 25}=\dfrac{100+81}{225}=\dfrac{181}{225}$
9	**B**	
10	**B**	a(b - c) +b(c – a) + c(a - b) = ab – ac + bc – ab + ac – bc = 0
11	**C**	
12	**A**	$\dfrac{x^9y^{-12}}{x^{-6}y^8}=x^9y^{-12}x^6y^{-8}=x^{9+6}y^{-12+-8}=x^{15}y^{-20}$
13	**A**	$\left(\dfrac{1}{2}\right)^{-3}=2^3=8$
14	**A**	$(a+2)^2 - (a-2)^2 = (a^2 + 4ab + 4) - (a^2 - 4ab + 4)$ $= a^2 + 4ab + 4 - a^2 + 4ab - 4 =$ **8ab**
15	**A**	$\dfrac{-2m^2-3mn}{-m}=\dfrac{-2m^2}{-m}-\dfrac{3mn}{-m}=2m+3n$
16	**C**	$\dfrac{(-4x^6)(-2x^4)}{-8x^{10}}=\dfrac{8x^{10}}{-8x^{10}}=-1$
17	**A**	$\dfrac{(x^2)(-xy)(-xz)}{-x}=\dfrac{x^4yz}{-x}=-x^3yz$
18	**D**	
19	**C**	

20	**D**	$2a - 2b - ax + bx = (2a - 2b) + (-ax + bx)$ $= 2(a - b) - x(a - b) = \mathbf{(a - b)(2 - x)}$
21	**D**	$x^2 + 4x - 12 = (x + 4)(x - 3)$ [a+b = 1, ab = -12. $\therefore$ a = 4, b = - 3]
22	**B**	$x^2 - 3x - 10 = (x - 5)(x + 2)$ [a+b = - 3, ab = -10. $\therefore$ a = - 5, b = 2]
23	**B**	$2x - 3 = 5 \Rightarrow 2x = 5 + 3 \Rightarrow 2x = 8 \Rightarrow x = \frac{8}{2} \Rightarrow x = 4$
24	**C**	$\frac{x}{2} = -22 \Rightarrow x = 2 \times -22 \Rightarrow x = -44$
25	**B**	$3z - 5 = 28 \Rightarrow 3z = 28 + 5 \Rightarrow 3z = 33 \Rightarrow z = \frac{33}{3} \Rightarrow x = 11$
26	**C**	$\frac{3}{x-2} = \frac{2}{x+3} \Rightarrow 3(x+3) = 2(x-2) \Rightarrow 3x+9 = 2x-4$ $\Rightarrow 3x - 2x = -4 - 9 \Rightarrow x = -13$
27	**C**	*Fractions have same denominator. So arrange the numerator in the ascending order.*
28	**B**	
29	**C**	
30	**A**	$\frac{(12x^2y^3)(6x^4y^2)}{3x^3y^3} = \frac{72x^6y^5}{3x^3y^3} = 24x^3y^2$